Total No. of printed pages = 7

3 (Sem 6) MTH M1

2015

MATHEMATICS

(Major)

Theory Paper : M-6.1

(Hydrostatics)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

(a) What will happen when there is an increase of pressure at any point of a liquid at rest under given external forces ?

(b) What do you mean by resultant vertical thrust on any surface of a homogeneous liquid at rest under the action of gravity ?

(c) What is surface of Buoyancy ?

(d) Define absolute zero of temperature.

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(e) What is thermal capacity of a body ?

(f) What is an adiabatic change ?

(g) If a body floats freely wholly or partially immersed in a fluid at rest under the action of gravity only, what are the vertical forces acting on the body ?

2. Answer the following questions : $2\times4=8$

(a) Obtain the differential equations of the lines of force at any point (x, y, z).

(b) Show that the positions of equilibrium of a body floating in a homogeneous liquid are determined by drawing normals from G, the centre of mass of the body, to the surface of buoyancy.

(c) If ρ_0 and ρ be the densities of a gas at $0°$ and $t°$ centigrade respectively, then establish the relation $\rho_0 = \rho(1+\alpha t)$ where $\alpha = \dfrac{1}{273}$.

(d) Show that a homogeneous liquid will be in equilibrium only when, the system of forces is conservative.

3. Answer any *three* parts : $5\times3=15$

(a) Determine the necessary condition that must be satisfied by a given distribution of forces X, Y, Z so that the fluid may maintain equilibrium.

(b) A hollow vessel containing some liquid, floats in a liquid. Determine the nature of equilibrium supposing that the body is symmetrical with respect to the vertical plane of displacement through its centre of mass and that the centre of mass of the body and that of the liquid contained are in the same vertical line.

(c) A circular area of radius a is immersed with its plane vertical and centre at a depth h ; find the depth of the centre of pressure.

(d) A mass of elastic fluid is at rest under the action of given forces. Determine the pressure at any point.

(e) Define a perfect gas. For a perfect gas establish the relation $C_p - C_v = R$, where the symbols have their usual notations.

4. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Prove that if the forces per unit of mass at (x, y, z) parallel to the axes are $y(a - z)$, $x(a - z)$, xy, the surfaces of equal pressure are hyperbolic paraboloids and the curves of equal pressure and density are rectangular hyperbolas.

(b) If the components parallel to the axes of the forces acting on the element of fluid at (x, y, z) be proportional to $y^2 + 2\lambda yz + z^2$, $z^2 + 2\mu zx + x^2$, $x^2 + 2\gamma xy + y^2$, show that if equilibrium is possible, we must have $2\lambda = 2\mu = 2\gamma = 1$.

(c) A mass of fluid rests upon a plane subject to a central attractive force μ/r^2, situated at a distance c from the plane on the side opposite to that on which is the fluid ; show that the pressure on the plane is

$\frac{\pi\rho\mu (a - c)^2}{a}$, a being the radius of the sphere of which the fluid, on the plane in the form of a cap, is a part.

(d) A mass M of gas of uniform temperature is diffused through all space, and at each point (x, y, z) the components of force per unit mass are $-Ax$, $-By$, $-Cz$. The pressure and density at the origin are P_0 and ρ_0 respectively. Prove that $ABC\,\rho_0\,M^2 = 8\pi^3 P_0^3$.

5. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) A quadrant of a circle is just immersed vertically with one edge in the surface, in a liquid, the density of which varies as the depth. Determine the centre of pressure.

(b) A vertical rectangle is exposed to the action of the atmosphere at a constant temperature. Assuming that the pressure exerted by the atmosphere varies with the height according to the usual law, determine the position of the centre of pressure of the rectangle.

(c) A hemispherical bowl is filled with water and two vertical planes are drawn through its central radius, cutting off a semi-line of the surface ; if 2α be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical, is $\tan^{-1}\left(\frac{\sin\alpha}{\alpha}\right)$.

(d) A right cone is totally immersed in water, the depth of the centre of its base being given. Prove that P, P′, P″ being the resultant pressure in its convex surface when the sines of the inclination of its axis to the horizontal are s, s′, s″ respectively.

$$P^2(s'-s'') + p'^2(s''-s) + p''^2(s-s') = 0.$$

6. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) A cone of given weight and volume floats with its vertex downwards, prove that the surface of the cone in contact with the liquid is least when its vertical angle is

$$2\tan^{-1}\left(1/\sqrt{2}\right).$$

(b) If the floating solid be a cylinder, with its axis vertical, the ratio of whose specific gravity to that of the fluid is σ, prove that the equilibrium will be stable, if the ratio of the radius of the base to the height be greater than $[2\sigma(1-\sigma)]^{1/2}$.

(c) Obtain an expression for the determination of height of a station by barometer when temperature is not constant.

(d) The height of the Torricellian vacuum in a barometer is *a* inches and the instrument indicates a pressure of *b* inches of mercury when the true reading is *c* inches. If the faulty readings are due to an imperfect vacuum, prove that the true reading corresponding to an apparent reading of *d* inches is

$$d + \frac{a(c-d)}{a+b-d}.$$