2014

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the moment of inertia of a circular disc of mass M and radius a about a diameter.

(b) State the perpendicular axes theorem on moments of inertia.

(c) Define the principal axes of a rigid body at a point O of the body.

(d) Write down the relation between the external torque and angular momentum of a rigid body.

(e) A particle moves on the surface of a sphere. What is the degree of freedom of the particle?

(f) A rigid body undergoes a rotation with angular velocity $\vec{\omega}$ about a fixed point O. If $\vec{v}$ is the velocity of any particle of the body having position vector $\vec{r}$ relative to O, state the relation connecting $\vec{v}$, $\vec{\omega}$ and $\vec{r}$.

(g) What do you mean by a conservative mechanical system?

2. Answer the following questions : 2×4=8

(a) The lengths AB and AD of the sides of a rectangle $ABCD$ of mass M are $2a$ and $2b$. Obtain the product of inertia of the rectangle about AB-AD.

(b) A rigid body of mass 2 units rotates with angular velocity $\vec{\omega} = (1, 1, -1)$ and has the angular momentum $\vec{\Omega} = (2, 3, -1)$. Find the kinetic energy of the body.

(c) A particle of mass 3 units is located at the point (2, 0, 0). The particle rotates about O with angular velocity $\vec{\omega} = \hat{k}$. Find the angular momentum of the particle about O.

(d) A particle moves under the influence of central force field $f(r)\vec{r}$ where $r = |\vec{r}|$, $\vec{r}$ being the position vector of the particle relative to the centre of force O. Show that the angular momentum of the particle about O is constant.

3. Answer the following questions : 5×3=15

(a) State the D'Alembert's principle and hence obtain the general equations of motion of a rigid body in vector form.

Or

A rough uniform board, of mass m and length $2a$, rests on a smooth horizontal plane, and a man of mass M, walks on it from one end to the other. Find the distance through which the board moves at this time.

(b) A solid homogeneous cone, of height h and vertical angle 2α, oscillates about a horizontal axis through its vertex; show that the length of the simple equivalent pendulum is $\frac{h}{5}(4 + \tan^2 \alpha)$.

Or

Obtain the Lagrangian for a simple pendulum and hence derive the equations of motion of the body.

(c) A body moves under the action of a system of conservative forces. Show that the sum of its kinetic energy and potential energy is constant throughout the motion.

4. The moments and products of inertia about three perpendicular axes OX, OY, OZ are given. Find the moment of inertia about any line through O. Hence show that the moment of inertia of a uniform cube about any axis through its centre is the same. 8+2=10

Or

Show that the moment of inertia of a right solid cone, whose height is h and the radius of whose base is a, is

$$\frac{3Ma^2}{20}\,\frac{6h^2+a^2}{h^2+a^2}$$

about a slant side, and $\frac{3M}{80}(h^2+4a^2)$ about a line through the centre of gravity of the cone perpendicular to its axis. 6+4=10

5. Define impressed forces and effective forces. A uniform rod OA, of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. 2+8=10

6. A uniform rod is held in a vertical position with one end resting upon a perfectly rough table, and when released rotates about the end in contact with the table. Find the motion. 10

Or

Show that the kinetic energy of a body moving in two dimensions is given by $T=\frac{1}{2}MV^2+\frac{1}{2}I\omega^2$ with usual meanings of the symbols. Also show that the angular momentum of the body about the origin O is given by

$$\vec{\Omega}=MVp\hat{n}+I\omega\hat{n}$$

where p is the perpendicular from O upon the direction of the velocity $\vec{V}$ of the centre of inertia and $\hat{n}$ is the unit vector normal to the plane of motion. 5+5=10

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