2014

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer *all* the questions : 1×7=7

(a) Why the motion of a planet relative to the sun is called two-body problem?

(b) Just mention how a spherical triangle is formed.

(c) Describe why lunar eclipse does not occur in every month at the time of full moon.

(d) What do you mean by circumpolar star?

(e) What is the difference between terrestrial and celestial longitudes and latitudes?

(f) Explain, whether refraction increases or decreases the zenith distance of a star. Give reasons.

(g) What is the relation between spherical and polar triangles?

2. Answer *all* the questions : 2×4=8

(a) Show that the section of a sphere by a plane is a circle.

(b) What do you mean by geocentric and annual parallaxes of a star?

(c) Discuss the effect of refraction on sunrise.

(d) Discuss how the coordinates—right ascension and declination of a star in celestial sphere are measured.

3. Answer any *three* from the following : 5×3=15

(a) Spell out five parts for a spherical triangle and prove that

sine (middle) = product of the tangents of the adjacents ($C = 90°$)

(b) Show that the velocity of a planet in its elliptic orbit is

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

where $\mu = G(M+m)$ and a is the semi-major axis of the orbit.

(c) Show that the time interval between the middle of a lunar eclipse and the time of opposition is

$$\frac{p\lambda}{(m-s)^2 + p^2} \text{ hrs (approx)}$$

where m and s are the hourly motions in longitude of the moon and the sun respectively, p is the hourly motion of the moon in latitude and λ is the moon's latitude at the instant of opposition.

(d) If a is the sun's altitude in the prime vertical at a place in latitude ϕ and L is its longitude, prove that

$$\phi = \sin^{-1}(\sin L \sin\varepsilon \operatorname{cosec} a)$$

(e) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi\sec\delta$, where the symbols have their usual meanings.

4. In a spherical triangle, prove that

$$\cos a\cos C = \sin a\cot b - \sin C\cot B$$

Also, in a spherical triangle if $b + c = \pi$, prove that $\sin 2B + \sin 2C = 0$. 6+4=10

5. State the Kepler's laws of planetary motion. Hence deduce the Kepler's laws from the Newton's single law of gravitation. 3+7=10

Or

What do you mean by refraction of a star? Discuss the effect of refraction on RA and declination of a star. 10

6. Show that the minimum angular distance D_0 of the moon and the sun for occurrence of solar eclipse should be $D_0 = \beta \cos j$ where

$$\tan j = \frac{\tan i}{1 - m}$$

the other symbols carry the usual meanings. 10

Or

What do you mean by equatorial horizontal parallax of a star?

Determine the geocentric parallax in right ascension and declination taking the earth as spheroid. 3+7=10

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