2014

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions

Group—A

(Mathematical Methods II)

(*Marks* : 35)

1. Answer the following questions : 1×4=4

(a) If $\vec{R}(u) = \frac{d}{du}\vec{S}(u)$, find $\int_a^b \vec{R}(u)\,du$.

(b) If the surface integral of $\vec{A}$ over a closed surface S vanishes, evaluate $\vec{\nabla}\cdot\vec{A}$.

(c) Write the transformation equations between Cartesian coordinates and spherical coordinates.

(d) Give the graphical representation of the Dirac delta function $\delta(x - x_0)$.

2. Answer the following questions : 2×3=6

(a) If $\vec{E} = -\vec{\nabla}\phi$, evaluate $\oint_C \vec{E} \cdot d\vec{r}$, where ϕ is a scalar function of r.

(b) Show that

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dV = \int_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi)\, d\vec{S}$$

(c) For an orthogonal curvilinear coordinate systems, show that

$$\hat{e}_2 = h_3 h_1\, \vec{\nabla} u_3 \times \vec{\nabla} u_1$$

where symbols stand for usual meanings.

3. Using Green's theorem in plane, find

$$\oint_C [(xy + y^2)\, dx + x^2\, dy]$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 5

Or

A fluid of density $\rho(x, y, z, t)$ moves with velocity $\vec{v}(x, y, z, t)$. If there is neither any source nor any sink, using divergence theorem, show that

$$\vec{\nabla} \cdot (\rho \vec{v}) + \frac{\partial \rho}{\partial t} = 0$$

4. Answer *either* (a) *or* [(b) and (c)] :

Either

(a) Prove that

$$\oint_C \vec{A} \cdot d\vec{\lambda} = \int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S}$$

where C is the curve bounding the surface S. Hence find $\oint \vec{r} \cdot d\vec{r}$. 8+2=10

Or

(b) Express $\vec{\nabla}\phi$ in the orthogonal curvilinear coordinate system. 7

(c) Find $\Gamma(-\frac{5}{2})$ provided $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. 3

5. Answer *either* [*(a)* and *(b)*] or [*(c)* and *(d)*] :

Either

(a) Prove that

$$\int_V (\vec{\nabla} \times \vec{B})\, dV = \int_S (\hat{n} \times \vec{B})\, dS$$

Where V is the volume enclosed by the surface S and $\hat{n}$ is the unit normal vector to the plane of dS. 6

(b) Show that

$$\int_{-\infty}^{+\infty} \delta(a-x)\,\delta(b-x)\,dx = \delta(a-b)$$ 4

Or

(c) Define Gamma function and establish

$$\Gamma(n+1) = n\,\Gamma(n)$$ 1+4=5

(d) Find the square of the elemental length in cylindrical coordinates and determine the corresponding scale factors. 4+1=5

GROUP—B

(Properties of Matter)

(*Marks* : 25)

6. Answer the following questions : 1×3=3

(a) Draw the stress-strain graph indicating the proportional limit and the yield point.

(b) State the principle on which the action of the split tip of a fountain pen's nib is based.

(c) A body is being moved horizontally through a viscous medium. What is the angle between the viscous force on the body and its weight?

7. Find the maximum length of a wire that can be suspended without breaking. Given that its breaking stress and density are $7 \cdot 2 \times 10^8 \text{ N/m}^2$ and $7 \cdot 2 \times 10^3 \text{ kg/m}^3$. Take $g = 10 \text{ m/s}^2$. 2

8. Answer any *two* of the following questions : 5×2=10

(a) A body is subjected to a stress. Show that the potential energy stored in the unit volume of the body is

$$\frac{1}{2} \times \text{stress} \times \text{strain}$$ 5

(b) Find an expression for an excess pressure at a point on a curved liquid surface. Hence find the excess pressure for a spherical soap bubble. 4+1=5

(c) A particle of mass m is moving through a viscous medium. If the viscous force varies linearly with instantaneous velocity v, find the expression for v as a function of time t. The initial velocity is v_0. 5

9. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] of the following questions :

Either

(a) Derive an expression for the twisting couple per unit twist of a rod of the length l, the radius r and the rigidity modulus η fixed at one end. 7

(b) Calculate the terminal velocity of an air bubble of radius 10^{-5} m rising in water of viscosity $10^{-3}\ \mathrm{Ns/m^2}$. Density of water is $10^3\ \mathrm{kg/m^3}$ and that of air is negligible. 3

Or

(c) Derive the relation

$$Y = 3K(1-2\sigma)$$

where Y, K and σ are Young's modulus, bulk modulus and Poisson's ratio respectively. 7

(d) Calculate the work done against surface tension in blowing a soap bubble from a radius of 10 cm to 20 cm, if the surface tension is $25 \times 10^{-3}\ \mathrm{N/m}$. 3

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