

2014

MATHEMATICS

(Major)

Paper : 2.1

(**Coordinate Geometry**)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) What will be the equation of the line $ax + by + c = 0$, if the origin is transferred to the point (α, β) ? 1
- (b) What is the locus represented by the equation $x^2 - 5xy + 6y^2 = 0$? 1
- (c) About which axis the parabola $y^2 = 4ax$ is symmetric? Justify your answer. 1
- (d) The parametric equations $x = a \sec \phi$, $y = b \tan \phi$ represent (i) an ellipse, (ii) a parabola, (iii) a hyperbola. 1
- Find the correct answer from above. 1

- (e) Write the relationship between the lengths of semi-major axis, semi-minor axis and the eccentricity for the standard equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad b > a \quad 1$$

- (f) What are the direction cosines of the normal to the plane given by the equation $2x - 4y + 3z = 9$? 1

- (g) Find the radius and centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$ 1

- (h) What is the general equation of a second-degree cone passing through the coordinate axes? 1

- (i) The shortest distance between the two lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

is given to be zero. What conclusion can you make about the lines? 1

- (j) What are the basic natures of the guiding curve and the generator for a right-circular cylinder? 1

2. (a) Prove that the equation

$$2x^2 - 5xy + 3y^2 - 2x + 3y = 0$$

represents two lines. 2

- (b) Prove that the equation

$$y^2 + 2ax + 2by + c = 0$$

represents a parabola, whose axis is parallel to the axis of x . 2

- (c) Find the equation of the plane through the point $(2, 3, 5)$ and parallel to the plane $2x - 4y + 3z = 9$. 2

- (d) Find the equation of the right-circular cone whose vertex is the origin, axis is the z -axis and semi-vertical angle is α . 2

- (e) The axis of a right-circular cylinder is $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{5}$ and its radius is 5. Find its equation. 2

3. (a) If by a rotation of the rectangular axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to

$$a'x'^2 + 2h'x'y' + b'y'^2$$

then show that

$$a + b = a' + b'$$

$$ab - h^2 = a'b' - h'^2 \quad 5$$

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- (b) Find the condition that the general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of straight lines. 5

Or

Prove that the straight lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will be equidistant from the origin if

$$f^4 - g^4 = c(bf^2 - ag^2)$$

4. (a) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes

$$9x - 7y + 6z + 48 = 0$$

$$x + y - z = 0 \quad 5$$

- (b) Prove that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. 5

(5)

Or

Find the length and equations of the line of the shortest distance between the lines

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$$

$$\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$$

5. Answer any four parts : 5×4=20

- (a) Find the equation of the pair of tangents from (x', y') to the parabola $y^2 = 4ax$.

- (b) If the line $\frac{lx}{a} + \frac{my}{b} = n$ cuts the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the ends of a pair of conjugate diameters, prove that $l^2 + m^2 = 2n^2$.

- (c) Prove that the middle points of chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ parallel to the diameter $y = mx$ lie on the diameter $a^2my = b^2x$.

- (d) Prove that from any point six normals can be drawn to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

(e) Prove that the equation of the polar of the origin with respect to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $gx + fy + c = 0$.

(f) Find the condition that the line $lx + my + n = 0$ is a tangent to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

6. Answer any four parts : 5×4=20

(a) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 = 49$ which passes through the line

$$2x + z - 21 = 0 = 3y - z + 14$$

(b) Show that the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$$

and guiding curve is $x^2 + 2y^2 = 1, z = 3$ is

$$3(x^2 + 2y^2 + z^2) + 8yz - 2zx + 6x - 24y - 18z + 24 = 0$$

(c) Prove that the section of a cone with vertex P and guiding curve the ellipse $z = 0, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by the y -plane is a rectangular hyperbola.

(d) Find where the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

meets the plane

$$2x + 4y - z + 1 = 0$$

(e) Find the equations of the tangent planes to the conicoid $2x^2 + 3y^2 - 4z^2 = 1$ which are parallel to the plane $x - 3y + z = 0$.

(f) Show that any normal to the conicoid

$$\frac{x^2}{pa+q} + \frac{y^2}{pb+q} + \frac{z^2}{pc+q} = 1$$

is perpendicular to its polar line with respect to the conicoid

$$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 1$$
